Test 3A - MTH 1410

Dr. Graham-Squire, Spring 2013

8:17
8:37

Name: Key

I pledge that I have neither given nor received any unauthorized assistance on this exam.

(signature)

Score breakdown on back

DIRECTIONS

1. Show all of your work and use correct notation. A correct answer with insufficient work or incorrect notation will lose points.
2. Clearly indicate your answer by putting a box around it.
3. Cell phones and computers are not allowed on the test. Calculators are allowed on the first — questions, but are not allowed on the last — questions of this test.
4. Give all answers in exact form, not decimal form (that is, put π instead of 3.1415, $\sqrt{2}$ instead of 1.414, etc) unless otherwise stated.
5. Make sure you sign the pledge.
6. Number of questions = 8. Total Points = 80.

80→72	3
72→64	4
64→56	5
56→48	6
48→40	7
40→	8

Need to spend more time on

* local Max/Min
* Tricky L'Hospitals
* Optimization
* Related Rates.

1. (12 points) Find the following limits: → Use correct notation!

(a) $\lim_{x\to\infty} \frac{\ln x}{x}$ → $\frac{\infty}{\infty}$

$\overset{H}{=} \lim_{x\to\infty} \frac{\frac{1}{x}}{1}$

$= \lim_{x\to\infty} \frac{1}{x} = \frac{1}{\infty} = \boxed{0}$

4

(b) $\lim_{x\to 0^+} \frac{\ln x}{x}$ → $\frac{-\infty}{0} = -\infty$

$\boxed{-\infty}$

2.5 1.5

4

(c) $\lim_{x\to\infty} 2x\sin\left(\frac{1}{x}\right)$ → $\infty \cdot 0$ $= \lim_{x\to\infty} \frac{\sin\frac{1}{x}}{\frac{1}{2x}}$ ✓

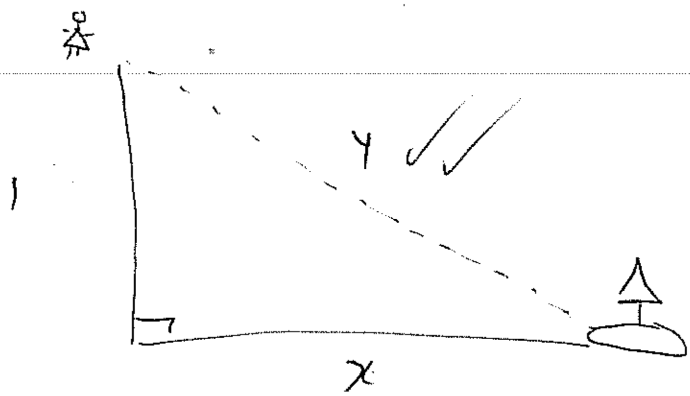
$\overset{H}{=} \lim_{x\to\infty} \frac{\cos\left(\frac{1}{x}\right)\cdot\frac{-1}{x^2}}{\frac{-1}{2x^2}}$ ✓ 0.5

4

$= \lim_{x\to\infty} \cos\left(\frac{1}{x}\right)\cdot\cancel{\frac{-1}{x^2}}\cdot\frac{2\cancel{x^2}}{\cancel{-1}}$ 0.5

$= 2\cos(0) = \boxed{2}$ ✓

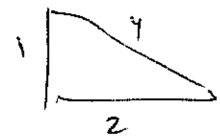
2. (10 points) Helen of Troy is standing atop a cliff rising one mile above the ocean. She is watching as one of the thousand ships launched for her face sails away from her. If the boat is 2 miles from the base of the cliff and is moving on the water away from her at a speed of 10 mi/hr, how fast is the (diagonal) distance between Helen and the boat increasing? Round your answer to the nearest 0.01 miles/hour.



$1^2 + x^2 = y^2$

$\frac{dx}{dt} = 10$

Want $\left.\frac{dy}{dt}\right|_{x=2}$



$\Rightarrow y^2 = 1+4$

$y = \sqrt{5}$

$\frac{d}{dt}\left(x^2+1=y^2\right)$

$2x\frac{dx}{dt} = 2y\frac{dy}{dt}$

$2(2)(10) = 2(\sqrt{5})\frac{dy}{dt}$

$\frac{40}{2\sqrt{5}} = \frac{dy}{dt}$

$\boxed{8.94 \text{ m/hr} = \frac{dy}{dt}}$

3. (10 points) Find the absolute maximum and absolute minimum of the function

4.5

$$f(x) = e^{x^3-5x^2+3x}$$

on the interval [0,5]. Round your answers to the nearest 0.01.

$f'(x) = e^{x^3-5x^2+3x} \cdot (3x^2-10x+3)$ ✓✓

$0 = e^{x^3-5x^2+3x}(3x^2-10x+3)$ ✓✓ $\quad e^{?}$ is never zero

$\Rightarrow 3x^2-10x+3=0$

$(3x-1)(x-3)=0$ ✓

$x = \frac{1}{3}$ or $x=3$ ✓

$f(0) = 1$

$f(1/3) = 1.6185$ ✓✓✓✓

$f(3) = 0.00012 \leftarrow$ min or 0.00

~~$f(5) = 3,270,000$~~

$f(4.5) = 29.224 \leftarrow$ max

29.22

4. (10 points) Use differentials to approximate the change in the surface area of a sphere when the radius is increased from 50 cm to 50.2 cm. Round your answer to the nearest 0.01 cm. The surface area of a sphere is given by $S(r) = 4\pi r^2$.

$dy = f'(s)dx$ ✓✓✓✓

$s = 50\text{cm}$ ✓

$dx = 0.2$ ✓

$f'(r) = 8\pi r$ ✓✓

$= 8\pi(50)\cdot(0.2)$ ✓

$= 80\pi \text{ cm}^2$ ✓

or $\boxed{251.33 \text{ cm}^2}$

Show your work!

5. (10 points) Consider the function $f(x) = \dfrac{\ln x}{x}$. It has the derivatives:
$f'(x) = \dfrac{1 - \ln x}{x^3}$ and $f''(x) = \dfrac{2\ln x - 1}{x^3}$.
Find the following:

(a) The x-value for all local maximums, if any exist.

(b) The interval(s) where the function is decreasing.

(c) The interval(s) where the function is concave ~~up~~. down.

(d) The x-value(s) of the inflection point(s), if any exist.

f not defined for $x \leq 0$.

$f'(x)$: $+$ $-$

0 e

$1 - \ln x = 0 \Rightarrow \ln x = 1 \Rightarrow x = e$

$f'(1) = +$

$f'(e^2) = -$

(a) local max at $x = e$

(b) decreasing on (e, ∞)

$f''(x)$: $-$ $+$

0 1.65

$2\ln x - 1 = 0 \Rightarrow \ln x = \frac{1}{2}$

$x = e^{1/2} = \sqrt{e} \approx 1.65$

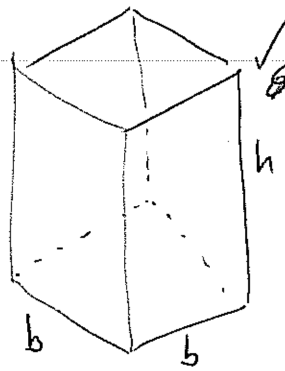
$f''(1) = -$

$f''(e) = +$

(c) concave ~~up~~ down on $(0, 1.65)$ $\sqrt{e}$

(d) inflection point at $x = 1.65 = \sqrt{e}$

6. (10 points) Postal regulations have the following stipulation for rectangular boxes that have a square bottom: the sum of the length of the base and the length of the height cannot exceed 10 feet. Find the <u>maximum</u> volume for such a box. Round your answer to the nearest 0.01 ft^3.



$b + h = 10 \qquad h = 10 - b$

$V = b^2 h$

$V = b^2(10 - b)$

$V(b) = 10b^2 - b^3$

$V'(b) = 20b - 3b^2$

$0 = b(20 - 3b) \qquad 20 - 3b = 0 \qquad 20 = 3b$

$\Rightarrow b = 0 \quad \text{or} \quad b = 20/3$

check

$+ \qquad -$

$0 \qquad 20/3$

$V'(0.1) = +$

$V'(7) = -$

$b = 20/3$ is a <u>max</u>

$V = \frac{20}{3} \cdot \frac{20}{3} \cdot (10 - 20/3) = \frac{4000}{27} \approx \boxed{148.15\ ft^3}$

7. (10 points) Use logarithmic differentiation to find the derivative of $y = (\sin x)^x$.

$$\ln y = \ln \sin x^{x}$$

$$\frac{d}{dx}\left(\ln y = x\left(\ln(\sin x)\right)\right)$$

$$\frac{y'}{y} = \ln(\sin x) + x\frac{1}{\sin x}\cdot\cos x$$

$$\boxed{y' = (\sin x)^{x}\left(\ln(\sin x) + x\cot x\right)}$$

8. (8 points) An astronaut on the moon throws a ball vertically upward. The height of the ball is given by the function

$$h(t) = 24t - 4t^2$$

where the height is given in feet, $t = 0$ is when the ball is released, and t is measured in seconds. Use calculus to solve an explain the following questions.

(a) When does the ball reach its maximum point? Use calculus to explain how you know it is a maximum.

(b) What is the highest point the ball reaches?

$h'(t) = 24 - 8t$

$0 = 24 - 8t \Rightarrow t = 3$

$+ \quad | \quad -$

3

(a) Max point at $t = 3$ seconds. Know it is a max b/c of sign chart

8

(or do $h''(t) = -8 \Rightarrow$ concave down.

(b) $h(3) = 72 - 36 = \boxed{36 \text{ ft}}$

8

Extra Credit(up to 3 points) Write either a 1 or a 3 into the space below to request how many points you want for extra credit. If you put a 1 you are guaranteed 1 point. If you put a 3 and less than half the class also puts a 3, then you get 3 points. If more than half the class puts a 3, you get zero.

1 : ~~||||~~ | 3 : ~~||||~~ ~~||||~~ ||

12	10	10	10	10	10	10	8	
L'Hosp 4/4/4	Rel Rates	Abs Max/Min	Diff'ential	Local Max/Min	Optimiz.	Log. diff	Ball ↗↘	
4, 2.5, 2.5	6.5	6	8	5	3	5.5	6	
4, 4, 2	4	4	2	2	5	7	6	
4, 2, 3.5	10	10	10	10	10	10	8	
2, 2, 2	10	10	9	3	8	9	8	
3, 2, 0	4	10	8.5	4	3	9	6	
4, 2, 2	8	9	10	9	8	10	6	
4, 2, 2	5	6	10	3	4	9.5	4	
2, 2.5, 1	3	8	5	5.5	3	7.5	6	
4, 3, 1	8.5	5	8	7	3	8	7	
4, 2, 1.5	5	9	10	1	9	4	6	
1, 1, 1	4	8	7	4	0	5	6	
4, 1.5, 1.5	9	6	10	5	6	7.5	6	
3, 2, 1	9.5	8	10	1	10	5.5	6	
2.5, 1, 2	4	10	10	6	9.5	7	6	
3, 2, 2	10	5	7	1.5	4	8.5	7	
4, 2.5, 2	5	10	4	4.5	6	8	8	
3, 1.5, 2	6	9.5	4	8	4	10	8	
4, 3, 2	6	10	10		9.5	7	6	
14, 3, 1	7	12	12	3	7	11	17	# over 75%